

Spherically symmetric gravitational collapse of a dust cloud in third order Lovelock Gravity

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We investigate the spherically symmetric gravitational collapse of an incoherent dust cloud by considering a LTB type space-time in third order Lovelock Gravity without cosmological constant, and give three families of LTB-like solutions which separately corresponding to hyperbolic, parabolic and elliptic. Notice that the contribution of high order curvature corrections have a profound influence on the nature of the singularity, and the global structure of space-time changes drastically from the analogous general relativistic case. Interestingly, the presence of high order Lovelock terms leads to the formation of massive, naked and time-like singularities in the 7D space-time, which is disallowed in general relativity. Moreover, we point out that the naked singularities in the 7D case may be gravitational weak therefore may not be a serious threat to the cosmic censorship hypothesis, while the naked singularities in the $D \geq 8$ inhomogeneous collapse violate the cosmic censorship hypothesis seriously.

I. INTRODUCTION

One of the most intriguing predictions of Einstein's general relativity is the existence of black holes which are formed from gravitational collapse in the last stage of heavy stars' life or in high-density regions of the density perturbations in the early universe [1]. The gravitational collapse of an incoherent spherical dust cloud is described by the Einstein

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equation $G_{ab} = \varepsilon(t, r)u_a u_b$ with the energy density $\varepsilon(t, r)$ and the time-like velocity vector u_a . In a reference frame comoving with the collapsing matter, it can be proved that the metric of spherically symmetric space-time reads [2]

$$ds^2 = -dt^2 + A(t, r)^2 dr^2 + R(t, r)^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where r is the comoving radial coordinate and t the proper time of freely falling shells. As well known, the solution of this metric is the Lemaitre-Tolman-Bondi (LTB) solution [2], which has been extensively studied not only in spherical collapse, but also in cosmology as well [3][4].

It is well known that the end state of gravitational collapse is a singularity of space-time curvature with infinite density of matter[5][6]. But, it is not known clearly whether such a singularity will be naked or covered by a event horizon. At the singularity, the laws of science and the ability to predict the future would break down. It led Roger Penrose to propose the (weak and strong) cosmic censorship hypothesis (CCH) [6][7], which to date remain unproven. The weak CCH asserts that there can be no singularity visible from future null infinity and light from singularity is completely blocked by the event horizon. It protects external observers from the consequences of the breakdown of predictability that occurs at the singularity, but dose nothing at all to observers who are cofalling with the collapsing massive body, while the strong CCH prohibits singularity's visibility by any observer. Despite almost 40 years of effort we still don't have a general proof of CCH [8]. On the contrary, it was shown that the seminal work of Oppenheimer and Snyder [1] is not a typical model and the central singularities formed in generic collapse are naked and observable [9][10].

Up to now, many quantum theories to unify all fundamental interactions including gravity have been proposed, the most promising candidate among them is superstring/M-theory. In string theory, extra dimensions were promoted from an interesting curiosity to a theoretical necessity since superstring theory requires a ten-dimensional space-time to be consistent from the quantum point of view [11][12]. It caused a renewed interest about the general relativity in more than 4 dimensions. Several LTB-like solutions to Einstein equations in higher dimensions have been obtained in recent years [13][14]. On the other hand, in recent years another renewed interest has grown in higher order gravity, which involves higher

derivative curvature terms. Among the higher curvature gravities, the most extensively studied theory is so-called Lovelock gravity [15], which naturally emerged when we want to generalize Einstein's theory in higher dimension by keeping all characteristics of usual general relativity excepting the linear dependence of Riemann tensor. In addition, Lovelock terms naturally occur in the effective low-energy action of superstring theory [16][17]. Hence, the Lovelock gravity provides a promising framework to study curvature corrections to the Einstein-Hilbert action. Since we are interested in what influence to the theory of gravity will be caused by Lovelock terms, for simplicity, we only consider pure Lovelock gravity and neglect the other terms in the low-energy expansion string theory. The Lagrangian of Lovelock theory is the sum of dimensionally extended Euler densities [15]

$$\mathcal{L} = \sum_{n=0}^m \alpha_n \mathcal{L}_n,$$

where arbitrary constants α_n are the Lovelock coefficients, and \mathcal{L}_n is the Euler density of a $2k$ -dimensional manifold

$$\mathcal{L}_n = \frac{1}{2^n} \delta_{c_1 b_1 \dots c_n b_n}^{a_1 b_1 \dots a_n b_n} R^{c_1 d_1}_{a_1 b_1} \dots R^{c_n d_n}_{a_n b_n}. \quad (2)$$

Here the generalized delta function $\delta_{c_1 \dots c_n}^{a_1 \dots a_n}$ is totally antisymmetric in both sets of indices and R^{cd}_{ab} is the Riemann tensor. Though the Lagrangian of Lovelock gravity consists of some higher derivative curvature terms, its field equations of motion contain the most symmetric conserved tensor with no more than two derivative of the metric. The \mathcal{L}_0 is assumed to be identity and c_0 the cosmological constant. \mathcal{L}_1 gives the usual Einstein-Hilbert action term. \mathcal{L}_2 is called Gauss-Bonnet term, it is a correction term of the action [18]. So far, the exact static and spherically symmetric black hole solutions in third order Lovelock gravity were first found in [19], and the thermodynamics have been investigated in [20][21][22].

It should be interesting to discuss the gravitational collapse in higher dimensional Lovelock gravity. The natural questions would be, for instance, how does the Lovelock terms affect final fate of collapse? What horizon structure will be formed? Whether solutions leads to naked singularities? Whether CCH can be hold? Recently, Maeda considered the spherically symmetric gravitational collapse of a inhomogeneous dust with the $D \geq 5$ -dimensional action including the Gauss- Bonnet term. He discussed its effects on the final fate of gravitational collapse without finding the explicit form of the solution [23]. Then, Jhingan and Ghosh considered the 5D action with the Gauss-Bonnet terms for gravity and give a exact

model of the gravitational collapse of a inhomogeneous dust [24][25]. It's interesting to investigate the effect of higher order terms of Lovelock gravity in gravitational collapse. we explore the gravitational collapse in comoving coordinates, and seek LTB-like solutions in the third order Lovelock gravity. The dimensions of space-time concerned is $D \geq 7$ because the third order term yields nontrivial effect in dimensions greater than or equal to 7 [26][27]. Using our solutions, we discuss the formation of singularities, and analyze the nature of them. In particular, we consider that whether such singularities would be hidden or be visible to outside observers. Since the CCH would be violate by naked gravitational strong singularities, we investigate the gravitational strength of naked singularities, and discuss that whether the naked singularities in third order theory is a serious threat to CCH.

This paper is organized as follows. In Sec II, for the $D \geq 7$ -dimensional space-time, we give the field equations in third order Lovelock gravity without a cosmological constant, and derive the LTB-like solutions. In Sec III, we investigate the final fate of the spherically symmetric gravitational collapse of a dust cloud. The subject of Sec IV is to analyze horizons (both apparent horizons and event horizons) and trapped surfaces, and explore that whether the singularity formed by gravitational collapse is hidden or visible to outside observers. The strength of the singularity is demonstrated in Sec V. Sec VI is devoted to conclusions and discussions.

Throughout this paper we use units such that $8\pi G = c^4 = 1$.

II. LTB-LIKE SOLUTIONS IN THIRD ORDER LOVELOCK GRAVITY

The action of third order Lovelock gravity in the presence of matter field can be written as

$$S = \int d^D x \sqrt{-g} (R + \alpha_2 \mathcal{L}_2 + \alpha_3 \mathcal{L}_3) + S_{matter}, \quad (3)$$

where α_2 and α_3 are coupling constants of the second order (Gauss-Bonnet) and the third order terms, respectively. In the low-energy limit of the heterotic string theory, α is regard as the inverse string tension and positive define [28]. Hence we restrict ourselves to the case $\alpha \geq 0$ in this paper. For future simplicity, we take coefficients $\alpha_2 = \frac{\alpha}{(D-3)(D-4)}$ and

$\alpha_3 = \frac{\beta}{72C_{D-3}^4}$. The Gauss-Bonnet term \mathcal{L}_2 is

$$\mathcal{L}_2 = R_{\mu\nu\sigma\kappa}R^{\mu\nu\sigma\kappa} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

and the third order terms of Lovelock Lagrangian is of the form

$$\begin{aligned}\mathcal{L}_3 = & 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}_{\mu\nu} + 8R^{\mu\nu}_{\sigma\rho}R^{\sigma\kappa}_{\nu\tau}R^{\rho\tau}_{\mu\kappa} \\ & + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^\rho_\mu + 3RR^{\mu\nu\sigma\kappa}R_{\mu\nu\sigma\kappa} \\ & + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^\sigma_\mu \\ & - 12RR^{\mu\nu}R_{\mu\nu} + R^3.\end{aligned}$$

Varying the action Eq. (3), we obtain the equation of gravitation field

$$G_{\mu\nu}^{(1)} + \alpha_2 G_{\mu\nu}^{(2)} + \alpha_3 G_{\mu\nu}^{(3)} = T_{\mu\nu}, \quad (4)$$

where

$$\begin{aligned}G_{\mu\nu}^{(1)} &= R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \\ G_{\mu\nu}^{(2)} &= 2(R_{\mu\sigma\kappa\tau}R_{\nu}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^\sigma_\nu + RR_{\mu\nu}) \\ &\quad - \frac{1}{2}\mathcal{L}_2g_{\mu\nu}, \\ G_{\mu\nu}^{(3)} &= 3R_{\mu\nu}R^2 - 12RR_{\mu}^{\sigma}R_{\sigma\nu} - 12R_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} \\ &\quad + 24R_{\mu}^{\alpha}R_{\alpha}^{\beta}R_{\beta\nu} - 24R_{\mu}^{\alpha}R^{\beta\sigma}R_{\alpha\beta\sigma\nu} \\ &\quad + 3R_{\mu\nu}R_{\alpha\beta\sigma\kappa}R^{\alpha\beta\sigma\kappa} - 12R_{\mu\alpha}R_{\nu\beta\sigma\kappa}R^{\alpha\beta\sigma\kappa} \\ &\quad - 12RR_{\mu\sigma\nu\kappa}R^{\sigma\kappa} + 6RR_{\mu\alpha\beta\sigma}R_{\nu}^{\alpha\beta\sigma} \\ &\quad + 24R_{\mu\alpha\nu\beta}R_{\sigma}^{\alpha}R^{\sigma\beta} + 24R_{\mu\alpha\beta\sigma}R_{\nu}^{\beta}R^{\alpha\sigma} \\ &\quad + 24R_{\mu\alpha\nu\beta}R_{\sigma\kappa}R^{\alpha\sigma\beta\kappa} - 12R_{\mu\alpha\beta\sigma}R^{\kappa\alpha\beta\sigma}R_{\kappa\nu} \\ &\quad - 12R_{\mu\alpha\beta\sigma}R^{\alpha\kappa}R_{\nu\kappa}^{\beta\sigma} + 24R_{\mu}^{\alpha\beta\sigma}R_{\beta}^{\kappa}R_{\sigma\kappa\nu\alpha} \\ &\quad - 12R_{\mu\alpha\nu\beta}R_{\sigma\kappa\rho}^{\alpha}R^{\beta\sigma\kappa\rho} - 6R_{\mu}^{\alpha\beta\sigma}R_{\beta\sigma}^{\kappa\rho}R_{\kappa\rho\alpha\nu} \\ &\quad - 24R_{\mu\alpha}^{\beta\sigma}R_{\beta\rho\nu\lambda}R_{\sigma}^{\lambda\alpha\rho} - \frac{1}{2}\mathcal{L}_3g_{\mu\nu}.\end{aligned}$$

The solution we search is collapse of a spherically symmetric dust in $D \geq 7$ space-time. Following LTB solutions, we assume that the system consists of a freely falling perfect fluid, it requires that the mean free path between collisions is small compared with the scale of

lengths used by observer. The energy-momentum tensor for the perfect fluid in comoving coordinates is

$$T_{\mu\nu} = \varepsilon(t, r)u_\mu u_\nu, \quad (5)$$

where $u_\mu = \delta_\mu^t$ is the velocity vector field. We assume that the energy density $\varepsilon(t, r)$ on the initial surface is smooth, that is, it can be extended to a C^∞ function on the entire real line. The metric in comoving coordinates is written in the form

$$ds^2 = -dt^2 + A(t, r)^2 dr^2 + R(t, r)^2 d\Omega_{D-2}^2, \quad (6)$$

where r is the comoving radial coordinate, and t is the proper time of freely falling shells. Plugging the metric Eq. (6) into Eq. (4), we have

$$\begin{aligned} G_r^t = & \frac{2-D}{A^5 R^5} (\dot{A}R' - A\dot{R}') [\beta(A^2 \dot{R}^2 + A^2 - R'^2)^2 \\ & + 2\alpha A^2 R^2 (A^2 \dot{R}^2 + A^2 - R'^2) + A^4 R^4] = 0, \end{aligned} \quad (7)$$

where an over-dot and prime denote the partial derivative with respect to t and r , respectively. Based on Eq. (7), we arrive at two families of solutions which satisfy

$$0 = \dot{A}R' - A\dot{R}', \quad (8)$$

and

$$\begin{aligned} 0 = & \beta(A^2 \dot{R}^2 + A^2 - R'^2)^2 \\ & + 2\alpha A^2 R^2 (A^2 \dot{R}^2 + A^2 - R'^2) + A^4 R^4, \end{aligned} \quad (9)$$

respectively. The second equation involves the Lovelock coupling constants, and leads to a trivial solution if $\alpha \rightarrow 0, \beta \rightarrow 0$. We will neglect it since we want to explore the Lovelock corrections to Einstein gravity. The solution of Eq.(8) reads

$$A(t, r) = \frac{R'(t, r)}{W(r)} \quad (10)$$

with an arbitrary function $W(r)$.

As it is possible to make an arbitrary re-labeling of spherical dust shell by $r \rightarrow g(r)$, we fix the labeling by requiring that, on the hypersurface $t = 0$, r coincides with the area radius

$$R(0, r) = r. \quad (11)$$

Apparently, every $t = \text{const}$ and $r = \text{const}$ slice of the space-time is a sphere of radius R . The radius R can be given an absolute significance by following interpretation. Considering two particles a and b distribution along the radial direction in comoving coordinates, the space distance between them at the coordinate time t can be obtained on the hypersurface $t = \text{const}$ as

$$\begin{aligned} l_{ab} &= \int_a^b dl = \int_a^b \sqrt{h_{ij} dx^i dx^j} \\ &= \int_{r_a}^{r_b} \frac{R'}{\sqrt{1+K}} dr = \int_{R_a}^{R_b} \frac{dR}{\sqrt{1+K}}, \end{aligned} \quad (12)$$

here h_{ab} is the induced metric on the hypersurface, $K(r) = W(r)^2 - 1$ and we assume $K(r) > -1$. In last step the fact $dR = R' dr + \dot{R} dt = R' dr$ at a constant t is used. Thus, the line element of actual space distance along radial direction is $\frac{dR}{\sqrt{1+K}}$. The signature of $K(r)$ corresponds to three types of solutions, namely hyperbolic, parabolic and elliptic, respectively. $K(r) = 0$ is the marginally bound case in which the metric takes the form of Minkowski metric on the hypersurface $t = 0$.

In comoving coordinates, the equations of momentum conservation $(T_i^\mu)_{;\mu} = 0$ are automatically satisfied, and the t -component reads

$$\begin{aligned} 0 &= -\frac{\partial \varepsilon}{\partial t} - \varepsilon \left(\frac{(R'^2)_{,t}}{2R'^2} + \frac{(D-2)(R^2)_{,t}}{2R^2} \right) \\ &= -\frac{\partial}{\partial t} (\varepsilon R^{D-2} R'), \end{aligned} \quad (13)$$

which gives the solution

$$\varepsilon(t, r) = \frac{\varepsilon(0, r) r^{D-2}}{R^{D-2} R'}. \quad (14)$$

Thus, the mass function is defined as

$$\begin{aligned} M(r) &= \frac{6}{D-2} \int \varepsilon(t, r) R^{D-2} dR \\ &= \frac{6}{D-2} \int_0^r \varepsilon(0, r) r^{D-2} dr, \end{aligned} \quad (15)$$

it is positive and increases with increasing r for the nonnegativity of energy density ε .

Based on Eq. (10) and $K(r) = W(r)^2 - 1$, the tt component of equations is given by

$$\begin{aligned} G_t^t &= \frac{2-D}{6R^{D-2}R'} \left[R^{D-7} \left(\beta(\dot{R}^2 - K)^3 + 3\alpha(\dot{R}^2 - K)^2 R^2 \right. \right. \\ &\quad \left. \left. + 3(\dot{R}^2 - K)R^4 \right) \right]' = -\varepsilon(t, r). \end{aligned} \quad (16)$$

Substituting Eq. (14) and Eq. (15) into Eq. (16), we get the real solution that

$$\dot{R}^2 = K + \frac{R^2}{\beta} \left[\left(\frac{\Theta + \Pi}{2} \right)^{\frac{1}{3}} + \left(\frac{\Theta - \Pi}{2} \right)^{\frac{1}{3}} - \alpha \right], \quad (17)$$

where

$$\begin{aligned} \Theta &= -2\alpha^3 + 3\alpha\beta + \beta^2\rho_D, \\ \Pi &= \sqrt{\beta^2(\beta^2\rho_D^2 - 4\alpha^3\rho_D + 6\alpha\beta\rho_D - 3\alpha^2 + 4\beta)} \end{aligned}$$

with $\rho_D = MR^{1-D}$. This equation governs the time evolution of R in D-dimensional third Lovelock gravity. For gravitational collapse, the solution of $R(t, r)$ takes

$$\dot{R} = -\sqrt{K + \frac{R^2}{\beta} \left[\left(\frac{\Theta + \Pi}{2} \right)^{\frac{1}{3}} + \left(\frac{\Theta - \Pi}{2} \right)^{\frac{1}{3}} - \alpha \right]}. \quad (18)$$

It is straightforward to check that other field equations are automatically satisfied when Eq. (17) is satisfied.

For arbitrary initial data of energy density $\varepsilon(0, r)$, the Eq. (18) completely specify the dynamical evolution of collapsing dust shells. In the general relativistic limit $\alpha \rightarrow 0, \beta \rightarrow 0$ with marginally bound case, Eq. (18) can be integrated to yield

$$t_b(r) - t(r) = \frac{2\sqrt{3}R^{\frac{D-1}{2}}}{(D-1)\sqrt{M(r)}}, \quad (19)$$

where $t_b(r)$ is an function of integration, and can be formulated as

$$t_b(r) = \frac{2\sqrt{3}r^{\frac{D-1}{2}}}{(D-1)\sqrt{M(r)}}.$$

Consequently, the function $R(t, r)$ is

$$R(t, r) = r \left[1 + \frac{D-1}{2\sqrt{3}r^{\frac{D-1}{2}}} t \sqrt{M(r)} \right]^{\frac{2}{D-1}}. \quad (20)$$

It is similar to the form of $R(t, r)$ in LTB and LTB-like-solutions [2][14].

We can consider that our LTB-like solution is attached at the boundary of the dust cloud, which is represented by a finite constant comoving radius $r = r_0 > 0$, to the outside vacuum region. The outside vacuum region is represented by the solution whose metric is

$$ds^2 = -F(\tilde{r})dT^2 + \frac{d\tilde{r}^2}{F(\tilde{r})} + \tilde{r}^2 d\Omega_{D-2}^2,$$

where $F(\tilde{r})$ takes form of

$$F(\tilde{r}) = 1 - \frac{\tilde{r}^2}{\beta} \left[\left(\frac{\tilde{\Theta} + \tilde{\Pi}}{2} \right)^{\frac{1}{3}} + \left(\frac{\tilde{\Theta} - \tilde{\Pi}}{2} \right)^{\frac{1}{3}} - \alpha \right], \quad (21)$$

where

$$\begin{aligned} \tilde{\Theta} &= -2\alpha^3 + 3\alpha\beta + \beta^2\tilde{\rho}_D, \\ \tilde{\Pi} &= \sqrt{\beta^2(\beta^2\tilde{\rho}_D^2 - 4\alpha^3\tilde{\rho}_D + 6\alpha\beta\tilde{\rho}_D - 3\alpha^2 + 4\beta)} \end{aligned}$$

with $\tilde{\rho}_D = \frac{6m}{(D-2)\Omega_{D-2}}\tilde{r}^{1-D}$. If we define that

$$\begin{aligned} m &= \frac{(D-2)\Omega_{D-2}M}{6}, \\ R &= \tilde{r}, \\ dT &= \frac{\sqrt{1+K(r)}}{F(R)}dt, \end{aligned} \quad (22)$$

we can prove that the LTB-like solution attached at a finite constant comoving radius $r = r_0$, where we represent this hypersurface as Σ , to the outside vacuum solution smoothly.

Proof: As seen from inside of Σ , the metric on Σ is obtained by

$$ds_\Sigma^2 = -dt^2 + R_\Sigma^2 d\Omega_{D-2}^2.$$

As seen from outside of Σ , it is

$$ds_\Sigma^2 = -(F(R_\Sigma)\dot{T}_\Sigma^2 - \frac{\dot{R}_\Sigma^2}{F(R_\Sigma)})dt^2 + R_\Sigma^2 d\Omega_{D-2}^2.$$

It can be checked that the induced metric is the same on both sides of the hypersurface Σ with the definition Eq. (22). It implies

$$F(R_\Sigma)\dot{T}_\Sigma^2 - \frac{\dot{R}_\Sigma^2}{F(R_\Sigma)} = 1. \quad (23)$$

We can define a function ζ by

$$\zeta \equiv \sqrt{F(R_\Sigma) + \dot{R}_\Sigma^2} = F(R_\Sigma)\dot{T}_\Sigma.$$

As seen from inside, the nonzero components of the extrinsic curvature \mathcal{K}_a^b of Σ are calculated as $\mathcal{K}_t^t = 0$ and $\mathcal{K}_i^i = \frac{\sqrt{1+K(r_0)}}{R_\Sigma}$. As seen from outside of Σ , we find $\mathcal{K}_t^t = \frac{\dot{\zeta}}{R_\Sigma}(F(R_\Sigma)\dot{T}_\Sigma^2 - \frac{\dot{R}_\Sigma^2}{F(R_\Sigma)})^{-1}$ and $\mathcal{K}_i^i = \frac{\zeta}{R_\Sigma}$. With the definition Eq. (22), \mathcal{K}_a^b is the same on both sides of Σ . It implies

$$\zeta = \sqrt{1+K(r_0)}. \quad (24)$$

Finally, we combine Eq. (23) and Eq. (24) giving the equation of motion for the hypersurface Σ as

$$\dot{R}_\Sigma^2 = 1 - F(R_\Sigma) + K(r_0).$$

It takes the same form as Eq. (29) with the definition Eq. (22). Thus, the LTB-like solution attached at a finite constant comoving radius $r = r_0$ to the outside vacuum solution smoothly.

□

III. SHELL FOCUSING SINGULARITY

In this section, we consider the final fate of the gravitational collapse in $D \geq 7$ space-time. As pointed earlier, in general relativity, a collapse leads to a singularity, and the conjecture that such a singularity must be covered by an event horizon is the weak CCH. There are two kinds of singularities: shell crossing singularity and shell focusing singularity which is defined by $R' = 0$ and $R = 0$, respectively. The characteristic of a singularity in space-time manifold is the divergence of the Riemann tensor and the energy density [6]. In our case, the Kretschmann invariant scalar $\mathcal{K} = R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}$ for the metric Eq. (6) is

$$\begin{aligned} \mathcal{K} = & \frac{2\ddot{R}'^2}{R'^2} + \frac{4(D-2)\ddot{R}^2}{R^2} + \frac{4(D-2)\dot{R}^2\dot{R}'^2}{R'^2R^2} \\ & + \frac{2(D-2)(D-3)\dot{R}^4}{R^4}. \end{aligned} \quad (25)$$

It can be certified that the Kretschmann scalar is finite on the initial data surface. According to Eq. (15), the energy density of fluid dust sphere is

$$\varepsilon(t, r) = \frac{(D-2)M'}{6R^{D-2}R'}. \quad (26)$$

Clearly the Kretschmann scalar and the energy density diverge when $R' = 0$ and $R = 0$. Hence, we have both shell crossing and shell focusing singularities.

Shell crossing singularities can be naked, but they are inessential. Although the mass density and curvature invariants blow up there, the metric is in fact continuous. This is seen from that the Riemann curvature tensor and the energy momentum tensor are well defined, if we replace the coordinates (t, r) by (t, R) . On the other hand, shell focusing singularities

are considered to be the genuine singularities in space-time manifold [9][29]. Henceforth, we only concern the shell focusing singularity here.

In third order Lovelock gravity, equations and solutions of gravitational collapse are quite different from counterparts in general relativity. Hence it is necessary to investigate whether the evolution of collapsing dust cloud leads to the formation of the shell crossing singularity. We assume each shell satisfies $\dot{R}(0, r) < 0$ initially. If $\dot{R} = 0$ is satisfied for some r after the initial moment, this shell ceases to collapse and then bounces ($\dot{R} > 0$), and the shell focusing singularity would not be formed. Indeed, it can be proven that in the case $\beta \geq 0$, $\dot{R}^2 - K \geq 0$ at the initial moment, the shell focusing singularity will be inevitably formed.

Proof: From Eq. (16), we find that

$$\frac{d(\dot{R}^2)}{dR} = \frac{(7-D)MR^{6-D} - 6\alpha\ell^2R - 12\ell R^3}{3\beta\ell^2 + 6\alpha\ell R^2 + 3R^4}, \quad (27)$$

where $\ell = \dot{R}^2 - K$. Clearly, $\frac{d(\dot{R}^2)}{dR} < 0$ if $\beta \geq 0$ and $\ell \geq 0$. Here we have used the condition $\alpha \geq 0$ which is mentioned in the last section. If ℓ is non-negative at the initial surface, it will be always non-negative as increasing \dot{R}^2 . Hence, $|\dot{R}|$ increase as decreasing R therefore the collapsing of the shell $r = \text{const}$ from the finite initial data $R(0, r) = r$ is accelerated. Thus, R will vanish in finite proper time for the comoving observer, the shell focusing singularity is inevitably formed. \square

Physically, the condition $l \geq 0$ is satisfied for $\beta \geq \alpha^2$. For $0 < \beta < \alpha^2$, one should take in mind Eq.(17) with a real solution, which gives a lower limit of ρ_D . Within this region, the numerical figure show the condition valid, but it is hard to analytically prove it. Moreover, one can show the condition $l \geq 0$ valid forever if it is satisfied at the beginning. Thus, the final fate of a freely falling fluid sphere, which have initial density $\varepsilon(0, r)$ and zero pressure, is a state of infinite energy density and curvature. In the rest of this paper we will only consider the case that the shell focusing singularity could be formed, and assume $\beta \geq 0$.

As demonstrate above, our solution can be attached to the outside vacuum solution at the boundary of the dust cloud. The outgoing property of the central singularity depends on the dominant term of $F(R)$ in vacuum solution for $R \rightarrow 0$ [34]. If the metric function $F < 0$, then the tortoise coordinate defined by

$$R^* = \int^R F^{-1} dR$$

has finite negative value, the singularity is space-like. If $F > 0$, the singularity is time-like.

If $F = 0$, the singularity is null since $|R^*| \rightarrow \infty$. From Eq. (21), we find that the metric function behaves around the central singularity as

$$F(R) \approx 1 - \left(\frac{M(r_b)}{\beta R^{D-7}}\right)^{\frac{1}{3}},$$

where r_b is the boundary of the collapsing dust cloud. Since we assume $\beta > 0$, if $D > 7$, we have $F(R) < 0$ hence the singularity is space-like. If $D = 7$ and $M(r_b) > \beta$, the singularity is also space-like. If $D = 7$ and $M(r_b) = \beta$, it is null. If $D = 7$ and $M(r_b) < \beta$, it is time-like. This conclusion is shown in the Penrose diagram FIG.1. The event horizon of such vacuum solution would be discussed in the next section.

If the energy density is independent on r , namely the energy density and the space are homogeneous, the time of the formation of the singularity t_{SF} is a constant, as in the general relativity case. A homogeneous space that is isotropic about some point is maximally symmetric and the curvature in such a space is a constant. In homogeneous case, the metric Eq. (6) takes the form as the Robertson-Walker metric [30][31]

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 + kr^2} + r^2 d\Omega_{D-2}^2 \right), \quad (28)$$

where k is a constant. In order to satisfy Eq. (11), the radial coordinate r can be normalized so that $a(0) = 1$. When the shell hits the shell focusing singularity, the time is completely determined by $a(t) = 0$. Considering metric Eq. (28) and $M(r) = \frac{6}{(D-1)(D-2)}\varepsilon(0)r^{D-1}$, we get

$$\dot{a}(t) = -\sqrt{k + \frac{a(t)^2}{\beta} \left[\left(\frac{\hat{\Theta} + \hat{\Pi}}{2}\right)^{\frac{1}{3}} + \left(\frac{\hat{\Theta} - \hat{\Pi}}{2}\right)^{\frac{1}{3}} - \alpha \right]}, \quad (29)$$

where

$$\begin{aligned} \hat{\Theta} &= -2\alpha^3 + 3\alpha\beta + \beta^2\hat{\varepsilon}, \\ \hat{\Pi} &= \sqrt{\beta^2(\beta^2\hat{\varepsilon}^2 - 4\alpha^3\hat{\varepsilon} + 6\alpha\beta\hat{\varepsilon} - 3\alpha^2 + 4\beta)} \end{aligned}$$

with $\hat{\varepsilon} = 6\varepsilon(0)a(t)^{1-D}/(D-1)(D-2)$. This equation does not contain the variable r , that is, $a(t)$ is a function of t independent on r . It implies that every shell will collapse into the shell focusing singularity at the same time. Obviously, this conclusion is hold in general relativity [1], Gauss-Bonnet gravity [24] and third order Lovelock case.

From the geometric perspective, it can be proven that in the homogeneous case the shell crossing singularity is ingoing null when $D = 7$ and is ingoing space-like when $D > 7$, as shown in FIG.1.

Proof: Considering Eq. (29) we have

$$\begin{aligned} \frac{6}{(D-1)(D-2)}\varepsilon(0) &= [\beta(\dot{a}^2 - k)^2 + 3\alpha(\dot{a}^2 - k)a^2 \\ &+ 3a^4](\dot{a}^2 - k)a^{D-7}. \end{aligned} \quad (30)$$

Since the energy density $\varepsilon(0)$ have a finite non-negative value, we find that the factor a behaves as $c(t - t_{SF})^{\frac{6}{D-1}}$ near the singularity $t = t_{SF}$, where c is a coefficient. We take the line element of the FRW solutions with the factor $a = c(t - t_{SF})^{\frac{6}{D-1}}$ to the conformally flat form as

$$ds^2 = a^2(t(\bar{t}, \bar{r}))b^2(r(\bar{r}))(-d\bar{t}^2 + d\bar{r}^2 + \bar{r}^2 d\Omega_{D-2}^2), \quad (31)$$

where

$$\begin{aligned} d\bar{t} &= \frac{dt}{a(t)b(r)}, \quad \bar{r} = \frac{r}{b(r)}, \\ b(r) &= \exp(\ln|r| + \ln|\frac{1}{r} + \sqrt{\frac{1}{r^2} + k}|). \end{aligned}$$

The range of \bar{t} for $t \in (-\infty, t_{SF})$ is $(-\infty, +\infty)$ when $D = 7$ and is $(-\infty, \bar{t}_0)$ for $D > 7$, where \bar{t}_0 is a constant. Thus, the shell focusing singularity is ingoing null for $D = 7$, while it is ingoing space-like for $D > 7$. \square

IV. HORIZON AND TRAPPED SURFACE

An important construction in general relativity is that of the trapped surface, which is indispensable in proving null-geodesics incompleteness in context of gravitational collapse [6]. In general relativity, the trapped surface is defined as a C^2 closed space-like two-surface that two families of null geodesics orthogonal to this surface are converging. Physically, it captures the notion of trapping by implying that if two-surface $\Gamma(t, r)$ ($t, r = \text{const}$) is a trapped surface, then its entire future development lies behind a horizon. The apparent horizon is the outermost marginally trapped surface for the outgoing null geodesics [32], therefore the trapped surface could not be formed during collapse without the occurrence of the apparent horizon. The main advantage of working with the apparent horizon is that it is local in time and can be located at a given space-like hypersurface. Instead, the event

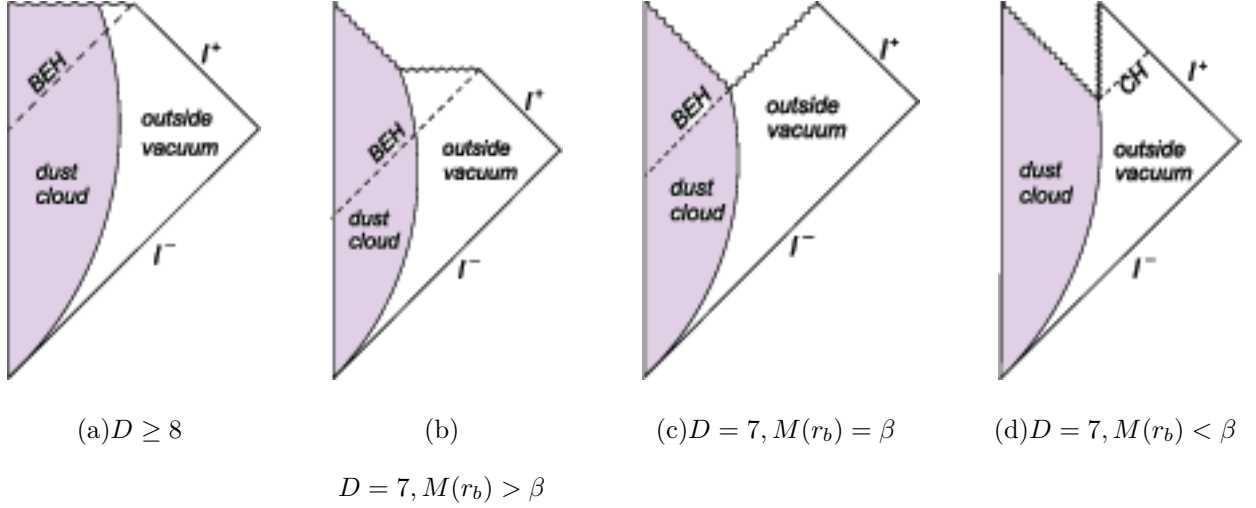


FIG. 1: Penrose diagram of the homogeneous collapse of a spherically symmetric dust cloud in third order Lovelock gravity. Zigzag lines represent the shell focusing singularities, $I^{+(-)}$ corresponds to the future (past) null infinity, BEH and CH stand for the black hole event horizon and the Cauchy horizon, respectively.

horizon coincide in case of static or stationary space-time, it is non-local. Moreover, in the vacuum region the apparent horizon coincides with the event horizon of the vacuum solution [9]. That is, without the presence of apparent horizons there is no event horizon.

Demanding the presence of the trapped surface in our spherically symmetric case implies

$$g^{\mu\nu} R_{,\mu} R_{,\nu} = -\dot{R}^2 + \frac{R'^2}{A^2} < 0, \quad (32)$$

and the condition for the existence of the apparent horizon with outward normals null is

$$g^{\mu\nu} R_{,\mu} R_{,\nu} = -\dot{R}^2 + \frac{R'^2}{A^2} = 0. \quad (33)$$

Using Eq. (10), the apparent horizon condition becomes

$$\dot{R}^2 - K = 1, \quad (34)$$

Combing Eq. (16) and Eq. (34), we obtain

$$R_{AH}(t_{AH}(r), r) = \sqrt{\frac{-3\alpha + \Psi}{6}}, \quad (35)$$

where

$$\Psi = \sqrt{12M(r)R_{AH}(t_{AH}(r), r)^{7-D} + 9\alpha^2 - 12\beta}. \quad (36)$$

Clearly, coupling constants α and β produces a change in the location of apparent horizons. Such a change could have a signification effect in the dynamical evolution of these horizons. It has been shown that in the 5D Gauss-Bonnet gravity case, positive α forbids apparent horizon from reaching the coordinate center thereby making the singularity massive and eternally visible [24], which is forbidden in the corresponding general relativistic scenario and $D \geq 6$ Gauss-Bonnet gravity [9][23][33]. In our case, positive α leads to noncentral naked singularities when $D = 7$. From Eq. (36), we find the condition that the apparent horizon is earlier than singular shell ($R_{AH}(t_{AH}(r), r) > 0$) for positive α in 7D case is $M(r) > \beta$. Oppositely, Eq. (36) could not be satisfied before the formation of the singularity in the 7D space-time in the case $M(r) < \beta$. Furthermore, the apparent horizon touches the singularity when $M(r) = \beta$. Thus, there is no shell can reach the apparent horizon if

$$M(r_b) < \beta, \quad (37)$$

with r_b the boundary of the dust cloud. It implies if the mass function $M(r_b)$ takes sufficiently small value, the shell focusing singularity is eternally visible from infinity during collapse, and leave open even the weak form of the CCH.

The condition of the formation of the eternally visible shell focusing singularity in 7D case is completely determined by the initial data of the energy density. With the help of Eq. (15), we can find the initial data of the energy density that condition Eq. (37) requires. For homogeneous case, such initial data satisfies

$$\frac{\varepsilon(0)r_b^6}{5} < \beta. \quad (38)$$

We can consider a more realistic model that $\varepsilon_0[1 - (\frac{r}{r_b})^n]$, which is a density profile where energy density decreases as an observer move away from the center, as is expected inside a star. This form of initial data leads to

$$\frac{n\varepsilon_0r_b^6}{5(n+6)} < \beta. \quad (39)$$

On the other hand, it is clear to see that in the $D \geq 8$ space-time, apparent horizons lies earlier than the singularity unless $r = 0$. Thus, the solution forbids the formation of the naked singularity except the singular point at the coordinate center, as same as in the general relativity and the Gauss-Bonnet gravity. As pointed by Christodoulou, the center singularity can be naked for suitable initial data of $\varepsilon(0, r)$ in the inhomogeneous case [9].

The physical picture captured here is that the coordinate center hits the singularity so early that other shells have not reached apparent horizons at this moment, hence the light from the singular coordinate center can escape to the infinity.

In Gauss-Bonnet gravity, the formation of the eternally visible shell focusing singularity is forbidden in $D \geq 6$ space-time, and is permitted when $M < \alpha$ and $\alpha > 0$ in the 5D case. Comparing results in second and third order Lovelock gravity, one can conjectures: for n order Lovelock gravity, such a naked singularity does not exist in $D \geq 2n + 2$ space-time, and would be formed when $M < \alpha_n$ in $D = 2n + 1$ case, where α_n is the coefficient of the highest order Lovelock term and is positive defined.

Now we consider the event horizon of the vacuum solution which describes the final fate of the collapse. Based on Eq. (21), we find the metric function $F(R)$ satisfies

$$M(r_b) = R^{D-7} \left[\beta(1 - F(R))^3 + 3\alpha(1 - F(R))^2 R^2 + 3(1 - F(R))R^4 \right].$$

The event horizon occurs when $F(R_{EH}) = 0$, thus we obtain

$$M(r_b) = R_{EH}^{D-7} (\beta + 3\alpha R_{EH}^2 + 3R_{EH}^4).$$

This equation leads to that

$$R_{EH} = \sqrt{\frac{-3\alpha + \sqrt{12M(r_b)R_{EH}^{7-D} + 9\alpha^2 - 12\beta}}{6}}.$$

Comparing this equation to Eq. (36), it is obvious to see the condition for the existence of the event horizon is the same to the condition for the presence of the apparent horizon, which is: it exist in the $D \geq 8$ case, and in the 7D case with $M(r_b) > \beta$. If $M(r_b) = \beta$ in 7D space-time, the event horizon touches the singularity at the center of the vacuum solution coordinate. This result is also shown in FIG.1. Thus, the $D = 7$ and $M(r_b) < \beta$ case indicates the existence of naked singularities which are eternally visible from infinity, and violates even the weak form of CCH. Such naked singularities are forbidden in standard general relativity, and is allowed in collapse in 5D second order Lovelock gravity with positive α [23]. Such conclusion coincides with the result which is obtained by investigating the apparent horizon and the outgoing property of the singularity.

V. STRENGTH OF SINGULARITY

In the case of formation of a naked singularity in gravitational collapse, one of the most significant aspects is the strength of such a singularity in terms of the behavior of the gravitational tidal forces in its vicinity [35]. The importance of the singularity strength lies in the fact that even if a naked singularity occurs, if it is gravitationally weak in some suitable sense, it may not have any physical implications and it may perhaps be removable by extending the space-time through the same [36]. The gravitational strength of a space-time singularity is characterized in terms of the behavior of the linearly independent Jacobi fields along the time-like or null geodesics which terminate at the singularity. In particular, a causal geodesic $\gamma(s)$, incomplete at the affine parameter value $s = s_0$, is said to terminate in a strong curvature singularity at $s = s_0$, if the volume three-form $V(s) = Z_1(s) \wedge Z_2(s) \wedge Z_3(s)$ defined as a two-form in the case of a null geodesic) vanishes in the limit as $(s \rightarrow s_0)$ for all linearly independent vorticity free Jacobi fields $Z_1(s), Z_2(s), Z_3(s)$ along $\gamma(s)$. A sufficient condition for a strong curvature singularity in 4D space-time is that in the limit of approach to the singularity, we must have along at least one causal geodesic $\gamma(s)$ [29][37],

$$\lim_{s \rightarrow s_0} (s - s_0)^2 \psi = \lim_{s \rightarrow s_0} (s - s_0)^2 R_{ab} V^a V^b > 0, \quad (40)$$

where V^a is the tangent vector to the geodesic. Essentially, the idea captured here is that in the limit of approach to such a singularity, the physical objects get crushed to a zero size, and so the idea of extension of space-time through it would not make sense, characterizing this to be a genuine space-time singularity. This condition has been applied to higher dimensional case by Ghosh, Beesham and Jhingan [38][24]. Here we follow them to assume that such condition can be applied to our case.

We consider radial time-like causal geodesics $U^\mu = \frac{dx^\mu}{d\tau}$ with the marginally bound case $K = 0$, here the affine parameter τ is the proper time along particle trajectories. According to such definition, U^μ satisfies $U^\mu U_\mu = -1$, that is,

$$-\left(\frac{dx^t}{d\tau}\right)^2 + R'^2 \left(\frac{dx^r}{d\tau}\right)^2 = -1. \quad (41)$$

Using the geodesic equation, we have

$$\frac{d^2 x^t}{d\tau^2} + \frac{1}{2}(R'^2)_{,t} \left(\frac{dx^r}{d\tau}\right)^2 = 0. \quad (42)$$

Substituting Eq. (41) into Eq. (42), we obtain that radial time-like geodesics must satisfy

$$\frac{dU^t}{d\tau} + \frac{\dot{R}'}{R'}((U^t)^2 - 1) = 0. \quad (43)$$

The simplest solution is the worldline of a freely falling particle, which is $U^\mu = \frac{dx^\mu}{d\tau} = \delta_t^a$. In terms of proper time we can describe it as

$$t_{SF}(r) - t = \tau_0 - \tau. \quad (44)$$

Eq. (37) shows that the naked shell focusing singularity in 7D space-time occurs when the region of the collapsing dust is sufficiently small. Moreover, as mentioned earlier, in $D \geq 8$ space-time only the central singularity can be naked. Hence, our purpose is to discuss the strength of the central singularity. We consider the expansion of $\varepsilon(r)$ near $r = 0$

$$\varepsilon(r) = \sum_{n=0}^{+\infty} \varepsilon_n r^n \simeq \varepsilon_0, \quad (45)$$

it specify that $R(t, r)$ behaves as $R_0(t)r$ (homogeneous case) near the coordinate center, where $R_0(t)$ is a function of t and vanishes at $t = t_{SF}$. Thus, we get

$$\psi = R_{ab}U^aU^b = -\frac{(D-1)\ddot{R}_0}{R_0}. \quad (46)$$

Clearly,

$$\lim_{\tau \rightarrow \tau_0} \frac{R_0(t)}{\tau - \tau_0} = \lim_{\tau \rightarrow \tau_0} \frac{R_0(t) - R_0(t_{SF})}{t - t_{SF}} = \lim_{\tau \rightarrow \tau_0} \dot{R}_0(t),$$

thus we have

$$\begin{aligned} \lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 &= \lim_{\tau \rightarrow \tau_0} \frac{R_0^2}{\dot{R}_0^2}, \\ \lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \psi &= \lim_{\tau \rightarrow \tau_0} -(D-1) \frac{R_0 \ddot{R}_0}{\dot{R}_0^2}. \end{aligned} \quad (47)$$

From Eq. (17), we obtain that

$$\begin{aligned} \lim_{\tau \rightarrow \tau_0} \dot{R}_0^2 &= \lim_{\tau \rightarrow \tau_0} \left(\frac{M_0}{\beta}\right)^{\frac{1}{3}} R_0^{\frac{7-D}{3}}, \\ \lim_{\tau \rightarrow \tau_0} \ddot{R}_0 &= \lim_{\tau \rightarrow \tau_0} \frac{7-D}{6} \left(\frac{M_0}{\beta}\right)^{\frac{1}{3}} R_0^{\frac{4-D}{3}} \end{aligned} \quad (48)$$

where M_0 is defined as $M_0 = \lim_{r \rightarrow 0} \frac{M(r)}{r^{D-1}}$ therefore has finite value. Substituting Eq. (48) into Eq. (47) we have

$$\lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \psi = \frac{(D-1)(D-7)}{6}, \quad (49)$$

the strong curvature condition Eq. (40) is not satisfied on the singularity near $r = 0$ in the 7D space-time, while it is satisfied in the $D \geq 8$ space-time.

Hence, the central shell focusing singularity is gravitational strong in the $D \geq 8$ space-time and may be gravitational weak in the 7D space-time since we chosen a special solution of Eq. (43). The difference between two cases is that the singularity can be eternally visible in the 7D case while it is forbidden in the $D \geq 8$ case. That is, the singularity is gravitational strong when the solution represents the black hole formation for arbitrary initial data. If a naked singularity is gravitational weak, it may not have any significant physical consequences so may not be a serious threat to the CCH. Thus, the naked singularity in the $D \geq 8$ case violates the CCH seriously, as in the general relativity [9][29], while the naked singularity may not be regarded as an essential counter example to the CCH in the 7D case.

Nevertheless, such a central singularity in the 7D case is gravitational strong in the general relativistic and second order Lovelock gravity. In the general relativistic limit $\alpha \rightarrow 0, \beta \rightarrow 0$, Eq. (40) takes the form

$$\lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \psi = \lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \left[\frac{(D-1)(D-3)M_0}{6R_0^{D-1}} \right],$$

with

$$\lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 = \lim_{\tau \rightarrow \tau_0} \frac{3R_0^{D-1}}{M_0}.$$

Thus,

$$\lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \psi = \frac{(D-1)(D-3)}{2} > 0, \quad (50)$$

the strong curvature condition is satisfied in $D > 3$ space-time. In second order Lovelock gravity limit $\beta \rightarrow 0$, we find that

$$\lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \psi = \frac{(D-1)(D-5)}{4} > 0, \quad (51)$$

the strong curvature condition is satisfied in $D > 5$ space-time. These results indicate that the Lovelock interaction weaken the strength of the singularity. Moreover, a generic expression for n order Lovelock gravity could be

$$\lim_{\tau \rightarrow \tau_0} (\tau - \tau_0)^2 \psi = \frac{(D-1)(D-2n-1)}{2n} > 0. \quad (52)$$

It shows that the strong curvature condition is satisfied in $D > 2n + 1$ space-time.

VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we have investigated the gravitational collapse of a spherically symmetric dust cloud in $D \geq 7$ space-time in third order Lovelock gravity without a cosmological constant. From field equations given by the third order Lovelock action, we discussed the solutions of three families $K(r) > 0$, $K(r) < 0$ and $K(r) = 0$. We discussed the final fate of the collapse, and gave a condition for the formation of the shell focusing singularity. We also analysed the global structure of the space-time and gave the Penrose diagram for the homogeneous case. It turns out that, the contribution of high order curvature corrections has a profound influence on the nature of the singularity, and the whole physical picture of the gravitational collapse changes drastically. High order Lovelock terms alters the course of collapse and the time of formation of singularities, modifies apparent horizon formation and the location of apparent horizons, and changes the strength of singularities.

The most attractive consequence is that an massive naked shell focusing singularity is inevitably formed in 7D space-time, which is quite different from that in the general relativity and in the Gauss-Bonnet gravity. However, as we shown, the strength of the naked singularity in 7D case is weaker than that in the general relativistic limit, therefore this may not be a serious threat to the CCH. On the other hand, unlike the 7D case, there is a serious threat to the CCH caused by the naked, gravitational strong singularity in the $D \geq 8$ inhomogeneous collapse, thus the CCH which is violated in general relativity could not be protected in Lovelock gravity.

When revising our paper in lines with reviewer's suggestions, we have received a paper by Ohashi, shiromizu and Jhingan[39] discussing the gravitational collapse in the Lovelock theory with arbitrary order which partially covers the result of present paper. In fact, they did not investigate the ingoing and outgoing properties of the singularity, the Penrose diagram, and whether the condition of strong singularity is satisfied. One may study such properties in a general Lovelock gravity similarly.

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